

The Football and Braking Distance: Model Data with Quadratic Functions

Algebra 2

Goals:

- Operate with numbers to solve problems. (1.01)
 - Simplify and perform operations with radical expressions.
 - Simplify expressions involving rational exponents.
- Operate with algebraic expressions to solve problems. (1.02)
 - Factor polynomials and other algebraic expressions completely over the real numbers.
- Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables. (3.02)
- Graph relations and functions and find the zeros of the functions. (3.03)
- Use quadratic equations and inequalities to solve problems. Solve by: (3.05)
 - Graphing.
 - Factoring.
- Find and interpret the maximum and minimum values and the intercepts of a quadratic function (3.06)
- Use equations which contain radical expressions to solve problems. Solve by: (3.11)
 - Graphing
 - Using properties of equality.
- Write and interpret an equation of a curve (quadratic) that models a set of data. (4.01)

Note: There are a large number of goals that are touched on in this lesson. Some are ones the students have been exposed to and some are ones that are ahead in the curriculum. The major focus of the lesson is on goals 5 and 7.

Materials and Equipment Needed:

- Copies of the handout for each student
- Graphing calculator
- Paper for note taking

Activity One: Football across the stadium

- Students enter the data given on the handout into lists on the calculator.

time(secs)	0.2	0.6	1	1.2	1.5	2	2.5	2.8	3.4	3.8	4.5
ht.(ft)	92	110	130	134	142	144	140	132	112	90	44

This data comes from page 110 of *Contemporary Precalculus through Applications*, by Barrett et al., Everyday Learning Corporation, copyright 2000.

- Look at a scatter plot of the data on the calculator. Discuss the appropriate window and meaning of the independent and dependent variables.
- Look at the animation that gives an illustration of the meaning of the data. Include focus of the questions of the day as the animation runs—when is the ball at 150 feet, 100 feet, more than 100 feet, 40 feet, when does it hit the ground?
- Using the quadratic regression on the stat calc menu, find an equation that will model the data. Enter this equation in Y1. Result: $y = -16.14x^2 + 64.37x + 79.78$, where x = time in seconds and y = height of football in feet.
- This equation models the height of the ball over time. Using this model, given a height, the time that the ball is at this height can be determined. (This is essentially solving a quadratic equation.) Use a horizontal line to represent the height (y -value) we seek. Then, on the calculator, use the intersect option under the CALC menu to find the point of intersection between the horizontal line and the prediction equation. Be sure to look at graphs and discuss whether an answer is possible and to determine the numeric value of the solution. Within the

context of the problem are these answers reasonable? Investigate the following height values using this technique.

- a. Begin with the consideration of a height of 150 feet. We look at the parabola with the graph of $y = 150$. The graphs show no solution. (May need to adjust window.) This investigation is attempting to solve the equation:
 $-16.14x^2 + 64.37x + 79.78 = 150$, which has no solution.
 - b. Find the time the ball has a height of 100 feet or $-16.14x^2 + 64.37x + 79.78 = 100$. There are two answers. Use intersect to find these. $(3.6456, 100)$ and $(0.3437, 100)$
 - c. Find the time(s) when the height is greater than 100 feet or
 $-16.14x^2 + 64.37x + 79.78 \geq 100$. Using the solutions from above we see a group of times that produce heights over 100 feet. $0.3436 < x < 3.6456$.
 - d. Find the time(s) the ball is at height of 40 or find the solution of
 $-16.14x^2 + 64.37x + 79.78 = 40$. Be sure to expand the window to show negative values of x . The graph shows two solutions, but within the context of the problem only one solution is feasible. There is no meaning to negative time. $(4.533, 40)$ and $(-0.544, 40)$.
6. When does the ball hit the ground? The ground has height 0 which means we are solving:
 $-16.14x^2 + 64.37x + 79.78 = 0$ or we are looking for the zeros on the graph. By expanding the window, two time-values can produce a height of 0, however, one is not reasonable within the context of the problem. We find the ball will hit the ground at time 4.982 seconds. The solution of -0.992 is not feasible. Use the zero option under CALC menu or intersect with $y = 0$.
 7. If the equation is simplified to not have whole-number coefficients, it becomes:
 $-16x^2 + 64x + 80 = 0$. This equation can be factored as $-16(x + 1)(x - 5) = 0$ and the zeros become visible in the factorization: $x = -1$ and $x = 5$. Talk about “real” problems not having pretty solutions. Look at the animation again to verify values.

Activity Two: Braking Distance

The following data is extracted from the graph on page 288 of the Glencoe Algebra II textbook.

speed of car (mph)	0	22	33	44	55
reaction distance (ft)	0	22	33	44	55
braking distance (ft)	0	19	43	79	128

1. Develop a mathematical model that can be used to predict the reaction distance given the speed.
2. Develop a mathematical model that allows us to predict braking distance given the speed of the car.
3. Develop a mathematical model that will predict total stopping distance given the speed of the car.
4. Usually when there is a wreck police investigators are attempting to determine the speed of the driver. To do this, the investigators measure the length of the skid marks left on the pavement. How could we use the information about the models to be able to predict the speed of the driver given the length of the skid marks?

1. The speed of the car is given in miles per hour. The conversion from mph to feet per second requires multiplying the speeds by $1.467 = 1\text{mph} \times \frac{5280\text{ft}}{60 \times 60\text{sec}}$. This helps students think about the time factor associated with the distances.
2. Using the lists on the calculator, put speed (in mph) in L1, reaction distance in L2, and braking distance in L3.
3. Looking at the data for L1, L2, the model is very straightforward: $y = x$. This tells us the reaction distance is equal to the speed. (Might want to discuss what this really means time wise. For example, it takes 0.68 seconds to react when traveling 55 mph. $55\text{mph} = 80.69$ feet per second. When going at the speed of 55 mph, it takes $55\text{ft} \cdot \frac{1}{80.69\text{ft/sec}} = 0.68\text{sec}$ to react.)
4. Look at the data for braking distance as a function of speed by graphing L1, L3. This is clearly quadratic and can be fit with quadratic regression to produce $l = 0.046s^2 - 0.199s + 0.264$. Superimpose this function over the data. Forecast several braking distances for high speeds using Y1 (100)=437 feet or Y1 (80)=277 feet.
5. Create a list that shows the total stopping distance for each speed by adding reaction and braking distances. This list can go in L4. Graph L1, L4. This data appears to be quadratic. When fit with the quadratic regression, the function produced is $l = 0.046s^2 + 0.801s + 0.264$ which is simply the sum of the two previous functions: $l = s$ and $l = 0.046s^2 - 0.199s + 0.264$. Makes sense!
6. What if we know the length of the skid marks is 200 feet. How do we determine the speed of the driver? We could attempt to solve the equation: $200 = 0.046s^2 - 0.199s + 0.264$ or we could show the graph of $l = 200$ with the graph of $l = 0.046s^2 - 0.199s + 0.264$ and find the point of intersection. Using the intersect option off the CALC menu on the calculator, we discover the two intersect at $s = 68.4$ which says the speed is 68.4 mph.
7. If you look in the book at page 288, the model used is $s = 2\sqrt{5l}$, where l is the length of the skid and s is the speed. Are we doing something different? If we take our equation $l = 0.046s^2 - 0.199s + 0.264$ and drop the constant and linear terms and simplify the coefficient of the quadratic term, we would have the model: $l = 0.04s^2$. If we solve for s , we get the equation $s = 2\sqrt{5l}$.

Follow Up Problem

For any 4-sided convex polygon, 2 distinct diagonals can be drawn. For any 5-sided convex polygon, 5 distinct diagonals can be drawn. For any 6-sided convex polygon, 9 distinct diagonals can be drawn. How many distinct diagonals can be drawn in a 20-sided polygon? Create the quadratic equation that will generate the number of distinct diagonals for n -sided polygons? (From Algebra II Indicators for Goal 4.02)

The data set that the students will develop is

sides	4	5	6	7	8	9
diagonals	2	5	9	14	20	27

The data is formed into ordered pairs (sides, diagonals). Students will probably recognize as they sketch in diagonals that from each of n vertices they can sketch $(n-3)$ diagonals and each diagonal connects 2 vertices. Hence the formula: $\frac{n(n-3)}{2}$ emerges. This is the same function that fits

the data: $f(x) = 0.5x^2 - 1.5x$, where x is the number of sides of the convex polygon.

Student Handout

The Football and Braking Distance: Model Data with Quadratic Functions Algebra 2

1. A student standing on the top of the bleachers throws a football across the field. The data that follows gives the height of the ball in feet versus the seconds since the ball was thrown.

time	0.2	0.6	1	1.2	1.5	2	2.5	2.8	3.4	3.8	4.5
ht.	92	110	130	134	142	144	140	132	112	90	44

This data comes from page 110 of *Contemporary Precalculus through Applications*, by Barrett et al., Everyday Learning Corporation, copyright 2000.

- Show a scatter plot of the data. What is the independent variable, and what is the dependent variable?
- What prediction equation (mathematical model) describes this data? (Use quadratic regression from the calculator.)
- When will the ball be at a height of 150 feet? Explain.
- When will the ball be at a height of 100 feet?
- At what times will the ball be at a height of greater than 100 feet?
- When will the ball be at a height of 40 feet?
- When will the ball hit the ground?

2. The following data is extracted from a graph on page 288 of the Glencoe Algebra II book.

speed of car (mph)	0	22	33	44	55
reaction distance (ft)	0	22	33	44	55
braking distance (ft)	0	19	43	79	128

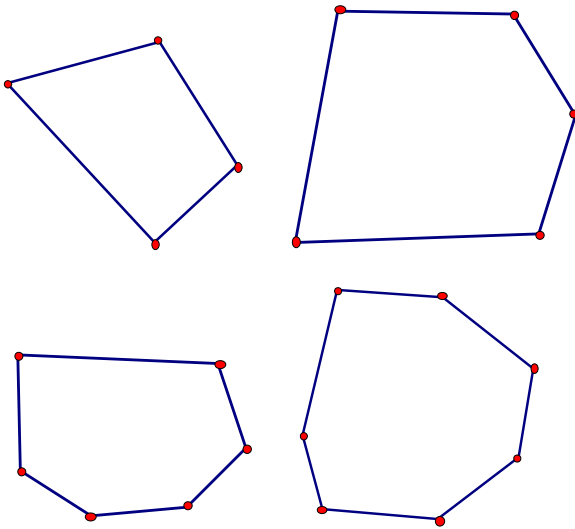
- Develop a mathematical model that can be used to predict the reaction distance given the speed.
- Develop a mathematical model that allows us to predict braking distance given the speed of the car.
- Develop a mathematical model that will predict total stopping distance given the speed of the car.
- Usually when there is a wreck police investigators are attempting to determine the speed of the driver. To do this, the investigators measure the length of the skid marks left on the pavement. How could we use the information about the models to be able to predict the speed of the driver given the length of the skid mark?

Follow Up Activity

Model Data with Quadratic Functions

Algebra II

1. For any 4-sided convex polygon, 2 distinct diagonals can be drawn. For any 5-sided convex polygon, 5 distinct diagonals can be drawn. For any 6-sided convex polygon, 9 distinct diagonals can be drawn. How many distinct diagonals can be drawn in a 20-sided polygon? Create the quadratic equation that will generate the number of distinct diagonals for n -sided polygons? (From Algebra II Indicators for Goal 4.02)
 - a. Use the geometric figures below to verify the first statements of the question and to add more data to the information.



- b. Form ordered pairs to represent this data. Think carefully about selection of which measurements are independent and which are dependent?
 - c. Can you add more data by just recognizing a pattern in the number of diagonals given the number sides or vertices? If so, add several more points (verify with a sketch).
 - d. Using Quadratic Regression, find an equation that models this data.
 - e. Because this is a mathematical idea, this data is fit perfectly by the Quad Reg. Using the meaning of the variables and the resulting equation, explain the geometry behind this formula.